Making an Investment/Production Decision in an Uncertain Environment

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Abstract — **This article presents the results of scientific research in the field of fuzzy data. Based on the developed methods, a model of making a decision on investment/production in conditions of uncertainty is described. This model includes several auxiliary models: a mathematical model of linear programming of investment project optimization, a model of the parametric α-level method of λ-continuation for the problem of fuzzy linear programming, an improved fuzzy five-factor Altman model. Conclusions and proposals of a scientific, theoretical and practical nature are also formulated.**

Keywords — enterprise creditworthiness assessment, Altman models, fuzzy sets, membership function, fuzzy measure, simulation modeling, decision-making under uncertainty, errors in accounting statements, business process optimization, optimal investment, investment under uncertainty.

I. Introduction

This In real economic conditions, the optimization question differs from standard linear programming problems in that the constraints are not set rigidly, and may be" slightly " violated due to inaccurately defined economic indicators or errors in the calculation. This situation is quite common and occurs when the exact values of the values on the right side of the constraints are unknown, and the lower guaranteed estimates or weighted averages of the input parameters are used instead. Also, if there is a need for strict fulfillment of the conditions, the resulting clear problem can lead to significantly biased point estimates of the found indicators or be unsolvable due to the emptiness of the allowed set. The resulting inaccuracies may be ignored when forecasting profit or loss, but there are areas of economic activity in which such "granules of information" are critical, for example, when evaluating working capital, where revaluation leads to the inability to conduct business, and underestimation – lost profits.

This problem can be solved by formalizing fuzzy concepts using the theory of fuzzy sets, which appeared as a result of generalization and reinterpretation of achievements in multivalued logic, probability theory and mathematical statistics, discrete mathematics, matrix theory, graph theory, and grammar theory. Expert systems built within the framework of this theory have proven themselves well not only in assessing quantitative, but also qualitative (linguistic) uncertainties, increasing the predictive function of models.

Methods for solving problems with fuzzy coefficients are related to the problem of constructing arithmetic operations on fuzzy sets and in linear programming use extended methods with different formal approaches. Currently, there is no general algorithm for solving such problems. The proposed partial approaches suggest performing volumetric calculations with fuzzy numbers, accompanied by the creation of a software package that is specific to each problem being solved.

Information systems that use fuzzy data, on the basis of the structure of the tasks to be solved, can be classified as partially structured, since only a part of their elements and the relationships between them are known. The research work is designed to reduce the developed information systems to structured ones.

Since the parameters of a fuzzy model can vary quite widely, the problem of the stability of the result to changes in the parameters is also extremely relevant.

The theory of solving problems of fuzzy mathematical programming is quite developed today [1]. The task set in this paper is to concretize the results obtained for their application in the conditions of uncertainty of economic indicators.

The solution of problems with fuzzy coefficients uses advanced linear programming methods, giving rise to the problem that the mathematical model under consideration must have an accurately expressed interpretation. But the constraints in the fuzzy form do not generate any clear set of possible values, and the fuzzy function cannot be maximized. Currently, there is no general algorithm for solving such problems. The proposed partial algorithms suggest performing volumetric calculations with fuzzy numbers, accompanied by the creation of a software package that is specific to each problem being solved [2]. Each new method and new representation of fuzzy numbers usually leads to the need to write new modules of software complexes [6]. Since the parameters of a fuzzy model can vary quite widely, the problem of the stability of the result to changes in the parameters is also extremely relevant. [11].

II. Methodology

We formulate the essence of the solved scientific problem of making a decision on investment/production in conditions of uncertainty, the purpose of which is to obtain models, methods and algorithms for making investment/production decisions and assessing creditworthiness in conditions of uncertainty [10].

It is necessary to solve the problem of modeling.

Given:

S-system (object of research-investment/production decision-making models);

X – multiple input parameters (m, n, $\tilde{c}_i$; $\tilde{a}_{ij}$ $\tilde{u}_j$, $\tilde{b}_j$ (i=1 m), (j=1, n));

Y – multiple output parameters (minimum cost of clay transportation, maximum revenue);

Z – multiple internal system parameters (x_{ij} – amount of clay (investment volume) m^3 / p (i=1,m), (j=1,n));

E – a set of environmental parameters (operating conditions-restrictions on investment volumes, volumes of clay for the plant);

Q – a set of indicators of the quality/efficiency of the system (improving the accuracy of calculations in conditions of uncertainty).

Substantive (verbal) statement of the scientific problem: to develop a model of the system S, which establishes the regularity of changes in the set of output parameters Y and the set of indicators of the quality/efficiency of the functioning of the system Q from the set of values of the input parameters X, the set of values of the internal parameters Z, the set of values of the parameters of the operating conditions E. At the same time, the values of the parameters of the sets X, Y, Z E are subject to the conditions of physical feasibility and economic conditions: X $\subseteq$ Hdop, Y $\subseteq$ Udop, Z $\subseteq$ Zdop, E $\subseteq$ Edop (where the index "dop" means "acceptable").

Formal statement of the scientific problem: $\mu : \rightarrow$ Y, Q $\vert$ X $\subseteq$ Xdop, Y $\subseteq$ Ydop, E $\subseteq$ Edop.

The model of optimal production/investment in conditions of uncertainty has the form:

1) if an enterprise considers m different investment projects and after n months it needs to get the maximum income (the enterprise has *m* quarries and *n* divisions and it needs to minimize transportation costs),

mathematical model of linear programming for optimizing investment projects is used [4, 5];

2) if it is necessary to solve the problem of improving the efficiency / quality of the system functioning because the input parameters are not clearly defined, for example $\tilde{c}_i$ – fuzzy revenue from the *i* project, i=1,m; $\tilde{a}_{ij}$ – fuzzy income/costs from the i-th project in the j-th year, i=1,m, $j = \overline{1, n}$; $\tilde{u}_j$ –fuzzy interest rate in the j-th year ($\tilde{a}_{ij}$ - m^3 of clay per day produced in the i-th quarry for the j-th plant, $\tilde{b}_j$ - m^3 of clay required by the j-th plant, $\tilde{c}_i$ – rubles, the cost of transporting one m^3 of clay from the i-th quarry (i=1 m) to the j-th plant (j=1, n)),

the model of the parametric α-level method of λ-continuation is used for the problem of fuzzy linear programming [7, 8];

3) if it is necessary to assess the possible risks from the point of view of the bankruptcy of the enterprise, **an improved fuzzy five-factor Altman model** is used [9, 10].

III. Acknowledgment

Based on the research conducted in the field of fuzzy data, conclusions and proposals of a scientific, theoretical and practical nature are formulated. The following original results are obtained:

1. As a result of solving the problems of optimal investment of projects by the enterprise, a new mathematical method for modeling optimal financing of investment projects was developed, which allows minimizing initial investments, maximizing the income received by the enterprise at the end of the investment process with an unchanged and periodically replenished investment fund of the enterprise, taking into account the risk index and the average duration of investment, in contrast to the existing one, is applicable for an arbitrary duration of investment [4, 5].

2. The proposed four-sided numerical parametric α-level method for obtaining the final results of the optimization problem is extended to the parametric α-level method λ - the continuation of the fuzzy linear programming problem. It is shown that the use of the algorithm of the parametric α-level method of λ - continuation of the fuzzy linear programming problem allows not only to add a level of uncertainty by expanding the confidence interval at the expense of α, but also to construct a method of inference for which the set of solutions will not be empty at sensitive coefficients due to the introduced гибкости-flexibility coefficient. The implementation of the proposed algorithm made it possible to eliminate the problem of unprofitability of production and reduce the cost of delivering clay to the Krasnodar Brick Factory by 30% by identifying unprofitable routes (50 thousand rubles instead of 80 thousand rubles per day). The optimal solution allows you to get the developed program in the MathCAD environment. For example, the minimum cost of transporting clay from quarries to factories is 50250 rubles per day. To do this, the first plant needs to deliver 200 m3 of clay from the second quarry and 250 m3 from the third quarry, and the second plant needs to deliver 300 m3 from the first quarry. At the same time, it is not cost-effective to deliver clay from the first quarry to the first plant, as well as from the second and third quarries to the second plant [7, 8].

3. A simulation model of the problem of fuzzy linear programming with the α-level method of λ–continuation is constructed, which allows us to check the adequacy of the developed mathematical model based on the data of a full-scale experiment (the standard deviation of the left-hand objective function is not more than 5%). This approach has an advantage over the existing one, since it allows you to estimate the probability of bankruptcy of an enterprise by entering fuzzy data, and also offers quantitative ways to formalize the fuzziness, giving the expert more information to make decisions in the conditions of sensitive problems, verifying the model by computational experiment [7, 8].

4. The Altman five-factor model is improved, extended by representing the input data as triangular fuzzy numbers. In the new block, when calculating the main

economic indicators based on accounting statements,the incoming values of the coefficients k_i (i=1,.., 5) are modeled, represented as fuzzy sets (based on expert opinions of economists). This representation of the coefficients is introduced in order to more fully take into account the fuzzy nature of the initial coefficients of the accounting statements. The implementation of the developed model eliminated the shortcomings of the four-factor Fox Z-model, giving adequate conclusions about the bankruptcy of the enterprise. The evaluation of the enterprise according to the improved Altman model allows even in critical zones of uncertainty (the result obtained is not included in any of the decision-making sets) to refer the enterprise to a well-defined area [7, 8].

5. Using the constructed simulation model, we tested the Altman model for assessing the creditworthiness of an enterprise using modern computer technologies, supplemented with fuzzy k_i indicators, which allows us to find the left-hand and right-hand sets of α-levels of the fuzzy set ki and observe how, with small changes in the coefficients, the probability of bankruptcy of the enterprise changes. This simulation shows how much the conclusion about the bankruptcy of the enterprise has not changed with an increase in the degree of uncertainty of the confidence interval. For example, for the Krasnodar brick factory, with an increase in the degree of uncertainty by 0.01, the conclusion about the probability of bankruptcy of the enterprise will not change. The root-mean-square deviation shows a rather small spread of all the main indicators (0.02) in the simulation model, which brings the simulation simulation closer to modeling using deterministic models [9].

6. An effective set of problem-oriented programs for conducting computational experiments based on the developed models, numerical methods and algorithms has been developed [10]:

- Risk-based Investment Portfolio Optimization Program (POIPR);
- Optimization program for the fuzzy linear programming problem of the parametric continuation method (POFLPP);
- Program for assessing the creditworthiness of the enterprise (PDMSC new).

The time spent on calculating the optimization problem of fuzzy linear programming with the α-level method of λ–continuation in the MathCad environment was reduced by 0.005 s. In comparison with the four-sided Fiedler problem.

Also, for the purpose of operational access, a mobile application for assessing the bankruptcy of an enterprise in Java in Android Studio has been developed.

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